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Auch mit seiner neuen Reihe „IAB-Discussion Paper“ will das Forschungsinstitut der Bundesagentur für Arbeit den Dialog mit der externen Wissenschaft intensivieren. Durch die rasche Verbreitung von Forschungsergebnissen über das Internet soll noch vor Drucklegung Kritik angeregt und Qualität gesichert werden.

Also with its new series "IAB Discussion Paper" the research institute of the German Federal Employment Agency wants to intensify dialogue with external science. By the rapid spreading of research results via Internet still before printing criticism shall be stimulated and quality shall be ensured.

Employment Effects of Centralization in Wage Setting in a Median Voter Model*

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Abstract

Most models on centralization in wage setting rest on the assumption of identical firms. This stands in sharp contrast to informal statements against centralization which rest on the argument that firms are heterogeneous and that equal treatment of firms by unions must therefore be inefficient. We analyse one aspect of this debate in the framework of a median voter model with heterogeneous firms but we don't find unique negative employment effects. Explorative investigations of the magnitude of the implied effects show however, that they are noteworthy only if differences between firms are large.

keywords: Bargaining, Centralization, Unemployment, Wages

JEL classification: J510, J520

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1 Introduction

This paper examines the employment effects of centralization in wage bargaining for an economy with heterogeneous firms. It is custom in the literature on centralization of wage bargaining to assume identical firms or to base models on one representative firm. This custom appears unrealistic and far away from the political debate on wage bargaining which is confined almost exclusively to differences between firms and regions. In a related note we show that firm heterogeneity may generate significant (but ambiguous) employment and welfare effects in a monopoly union model. The result is a straightforward application of ‘ancient’ results from monopoly theory dating back to Robinson (1933).¹ A weakness of the model is, however, that it rests upon a somewhat *ad hoc* specification of the union utility function $U = \sum_i u_i(w_i, n_i)$, i.e. the central union utility is the (possibly weighted) sum of individual union (or workforce) utility functions. Here we replace this *ad hoc* specification by another one: The median voter mechanism. Median voter models rest on the additional assumption of single peakedness of worker utility functions and thus are in principle applicable only as long as wages are the only concern of workers.² Therefore they are even more restrictive. But this restrictiveness comes together with a microfoundation which allows us to derive somewhat more intuitive conditions for the existence of negative or positive employment effects.

We anticipate that the employment effects are not unique. We are able to state some general properties of utility and production functions which are responsible for the direction of the employment effects, but this is only possible at the cost of realism, i.e. we have to confine the analysis to a very simple setting with two firms (or branches) only. An important shortcoming of the ‘general’ statements is that they are hardly testable empirically. Therefore the model leaves many questions unanswered.

The plan of the paper is as follows: In the following section we present a general formulation of the model. Then we try to obtain some general statements regarding employment effects of centralization. Since the general statements are rather weak and give us no clue about the magnitude and relative importance of the effects, we illustrate them in some small simulation

¹For a general analysis see Shih, Mai, and Liu (1988). Their results are derived in a goods market monopoly framework but applicable to union bargaining after straightforward relabelling.

²For an extensive discussion see e.g. Hoel, Moene, and Wallerstein (1993). We will provide a short discussion of the problems involved in the conclusion.

experiments. We conclude the paper with a short discussion of the central model assumptions.

2 A non-Technical Sketch of the Main Driving Forces

Since the details of the model are rather technical and involved we start by presenting the main idea with a graphical illustration. Consider a world with two firms, each with linear inverse labour demand curve $w(n)$. We start with equilibrium w_0, n_0 . What will happen with gross employment if labour demand of firm 1 is shifted downwards by θ ? A central union setting the same wage in each firm will generate no employment effect since the decrease of n_1 is exactly offset by the increase of n_2 . As long as we do not obtain a corner solution (labour demand of firm 1 is zero) the value \bar{w} set by the union does not matter.

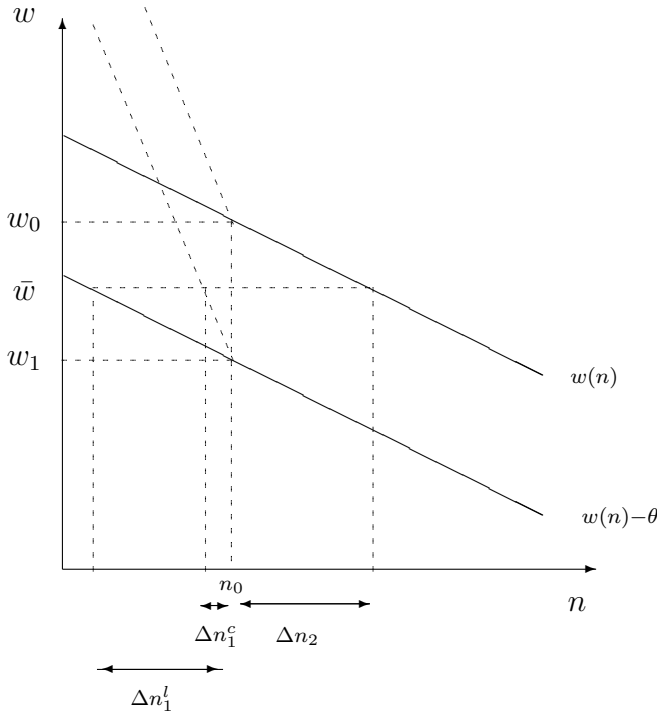


Figure 1: ω_i and Ω

To see our centralization effect now imagine that labour demand curves are convex. For simplicity we assume that they are piecewise linear, i.e. that they

are steeper on the left hand side from n_0 (dashed lines). Now the employment loss in firm 1 is considerably lower than the increase in firm 2 for $w = \bar{w}$. As can be seen easily from the picture, net employment change may even be negative if the central union does not respond to the shock by lowering the wage. However, there remains a large range of centralised wages with positive net employment changes. In the following rather technical sections we try to determine the wage response of a democratic centralised union. And we find that it is large enough to obtain a positive net employment change. Of course, our results apply also to the opposite case with concave labour demand. Then we obtain negative employment changes.

3 The Technical Details of the Model

The framework of our voting model follows Blair and Crawford (1984). Blair and Crawford investigate the conditions for the existence and uniqueness of a voting equilibrium in union worker decisions. To clarify things in the employment and welfare analysis later on, our notation is a little bit more fussy than theirs.

Labour demand of firm i is

$$n_i = \max\{0, \phi_i(w_i) + \theta_i + \xi_i\}$$

with $\phi'_i(w_i) < 0$.³ θ_i denotes a random disturbance term which is revealed after bargaining has taken place whereas ξ_i is known before contracting. Note that the additive form $\phi_i(w_i) + \theta_i$ implies a shock having no effect on labour demand and technology parameters. Therefore additiveness of shocks is plausible in the short run, since for this period the Leontieff technology is a good approximation to reality. This argument is much weaker for ξ , since ξ represents (at least) medium run heterogeneity between firms which has more structure in reality (e.g. represented by differences in production function parameters.)

³The max operator is introduced here to handle the possibility that $\theta_i < -\phi_i(w_i) - \xi_i$. This saves us to restrict the range of θ_i . Blair and Crawford are a little bit sloppy here. They omit the max operator and point to the fact that “the assumption of an additive Error can lead to negative labor demand, a situation that is clearly impossible. This specification was chosen largely for expositional convenience.” With the max operator the obvious interpretation is that the firm is shut down (i.e. employment of the firm is zero) with strictly positive probability.

The max operator eliminates the possibility that demand could become negative for sufficient small values of θ_i and ξ_i . The interpretation is straightforward: If $\phi_i(w) < \theta_i + \xi_i$, the firm closes down. Depending on the Distribution of ξ_i , there is a positive probability that this happens. At this stage of our analysis we take ξ_i as given (deterministic).

To employ the median voter theorem for our analysis, we have to check whether the expected utility functions of all workers are single-peaked. To this aim consider the utility maximization problem of a worker with seniority⁴ s and von Neuman-Morgenstern expected utility function

$$E[U(w|s)] = u(w)P[n > s] + u(b)P[n < s]$$

with wage w and alternative income level b which is assumed to be exogenous in our simple setting. $u(w)$ is assumed to be twice continuously differentiable with $u'(w) > 0$ and $u''(w) < 0$, i.e. workers are risk-averse.⁵ Usage of the seniority index s implies that we assume the existence of a unique ordering of all workers (including unemployed ones) prescribing in which order employees are dismissed if labour demand decreases.⁶

After substitution of n we can write the probability that the worker becomes unemployed $P(n < s)$ as $F_i(s - n_i(w) - \xi_i)$ where $F_i(\cdot)$ is the cumulative distribution function of θ_i (without loss of generality we can set the expectation of θ_i to zero⁷). Then a more explicit expression of the expected utility is

$$E[U(w|s)] = u(w)\{1 - F_i(s - \phi(w) - \xi_i)\} + u(b)F_i(s - \phi(w) - \xi_i)$$

with first and second order conditons

$$\frac{\partial}{\partial w}E[U(w|s)] = u'(w)\{1 - F(\bar{\theta}_i)\} + \phi'(w)f(\bar{\theta})\{u(w) - u(b)\} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial^2}{\partial w^2}E[U(w|s)] &= 2f(\bar{\theta}_i)u'(w)\phi'(w) + (1 - F(\bar{\theta}_i))u''(w) \\ &\quad + (u(w) - u(b))\{f(\bar{\theta}_i)\phi''(w) - f'(\bar{\theta}_i)\phi'(w)^2\} < 0 \end{aligned} \quad (2)$$

⁴We assume that firms rank workers according to seniority tenure). The most recently employed is dismissed first (if labour demand decreases) and so on. Of course, the ranking may be based on other criteria, for example on the reliability of workers or their position in the firm.

⁵Note that we deviate here from Blair and Crawford (1984) by removing the index relating to $u(\cdot)$. We do this for convenience (since even then the model contains more heterogeneity in the model than we can handle).

⁶We use the term 'seniority' in a metaphorical manner, since seniority is not the only criterion commanding dismissal. For unemployed workers it is not applicable at all. However, other properties of workers may substitute seniority, for example productivity differences not reflected by remuneration. For a discussion of the problems associated with a seniority index see Blair and Crawford (1984), Grossman (1983), Burda (1990).

⁷This is so because n_i is shifted by ξ .

where $\bar{\theta}_i := s - \phi(w) - \xi_i$. Blair and Crawford (1984) show (by setting (1) to zero and straightforward manipulation) that $E[U(w|s)]$ has a unique maximum if the inverse mill's ratio

$$\frac{f_i(\bar{\theta})}{1 - F(\bar{\theta})} \quad (3)$$

is increasing and the expression

$$\frac{-u'(w)}{\phi'_i(w)(u(w) - u(b))} \quad (4)$$

is decreasing in w . We do not try to present an exhausting analysis of the conditions necessary to guarantee single-peakedness of expected utility here but assume here simply that they are met.⁸

By setting the derivative in (1) to zero, we obtain the preferred wage, call it $\omega(\xi, s)$ of a worker with seniority s as an implicit function of the parameters b , ξ_i and parameters of the distribution function $F(\theta)$. Let us pause here for a moment to derive some results on the derivatives and shape of ω . The derivatives $\partial\omega/\partial s$ and $\partial\omega/\partial\xi$ are of central interest in our context. $\partial\omega/\partial s$ is obtained by implicit differentiation

$$\frac{\partial\omega}{\partial s} = -\frac{\partial^2 E[U]/\partial w \partial s}{\partial^2 E[U]/\partial w^2}$$

Since $\partial^2 E[U]/\partial w^2$ must be negative (by utility maximization), the sign of $\omega'(s)$ is equal to the sign of the numerator

$$\frac{\partial^2}{\partial w \partial s} E[U(w|s)] = \phi'(w)\{u(w) - u(b)\}f'(\bar{\theta}) - u'(w)f(\bar{\theta})$$

To show that this is negative we substitute $\phi'(w)$ from (1) to obtain

$$\frac{\partial^2}{\partial w \partial s} E[U(w|s)] = -u'(w) \left\{ \frac{f'(\bar{\theta})}{f(\bar{\theta})} (1 - F(\bar{\theta})) + f(\bar{\theta}) \right\}$$

As noted above, single-peakedness requires the inverse mills ratio (3) to be an increasing function of w . Substitution of this condition, i.e.

$$\begin{aligned} \frac{d}{dw} \left[\frac{f(\bar{\theta})}{1 - F(\bar{\theta})} \right] &= -\phi'(w) \frac{f(\bar{\theta})^2 - \{1 - F(\bar{\theta})\}f'(\bar{\theta})}{\{1 - F(\bar{\theta})\}^2} > 0 \\ \Leftrightarrow f(\bar{\theta})^2 - \{1 - F(\bar{\theta})\}f'(\bar{\theta}) &> 0 \end{aligned}$$

⁸Again we refer to the relevant literature Blair and Crawford (1984), Grossman (1983), Burda (1990).

into the expression in curly braces above gives the (in no respect surprising) result. Since ξ appears like s inside $f(\cdot)$ and $f'(\cdot)$ but with opposite sign, $d\omega/d\xi > 0$ by the same argument. Below we will find that the second derivatives of ω play an important role in the evaluation of centralization or decentralization. Their signs are, however, undetermined without further restrictions on f , ϕ and u . This can be shown after tedious manipulations. To keep the text paper readable the details are shifted to the appendix.

We want to use this framework now in order to assess employment and welfare effects of centralization in wage bargaining. For simplicity we consider an economy where firms do not compete for workers, i.e. labour demand functions are independent of each other. Though this is an extreme case applying only when firms are far away from each other and worker mobility is small (or labour is differentiated in some other way, for example qualification) this assumption gathers an essential feature of labour markets: In labour markets with perfect competition the centralization debate⁹ were senseless, since the market would force firms to pay equal wages and unions loose the ability to differentiate wages.¹⁰

In our simple economy central wage setting occurs if all workers in the economy vote for one single wage, whereas local wage setting takes place when only workers in the employment pool (region/branch) of each firm vote for a wage applying to this firm. As will become clear below, the comparison of central and local bargaining outcomes is quite involved for models with more than two firms and general forms of firm heterogeneity. Therefore we confine our analysis to the simplest case with two firms only and additive stochastic heterogeneity. Though this is a serious limitation, it allows us to gather some first insights into the structure of the problems.

4 Some Analytic Results

If wages are set locally, we obtain the median wage w_i in firm $i \in \{1, 2\}$ simply by setting s_i to $q_i/2$ where q_i is the mass of the employment pool related to firm i :

$$w_i = \omega_i(\xi_i, q_i/2)$$

⁹To state it more precisely: the debate referring to firm heterogeneity.

¹⁰The fast growing current literature on thin labour markets backs up this view. See e.g. Bhaskar and To (1999a, 1999b), Bhaskar, Manning, and To (2002), Manning (2002), Lewis (1986).

If the workers in both pools vote for a common wage claim, the median worker index \tilde{s} is implicitly defined by the equation

$$\omega_1(\xi_1, \tilde{s}) - \omega_2(\xi_2, (q_1 + q_2)/2 - \tilde{s}) = 0.$$

We assume that the pool sizes q_1 and q_2 and the ranges of the heterogeneity parameters ξ_i are such that $\tilde{s} \in [0, q_1]$ to eliminate ‘degenerate’ special cases here (the general case is discussed briefly in the appendix). Figure (4) illustrates the relation between local and central median wages.

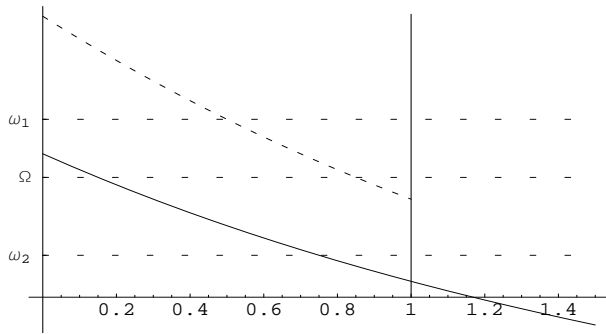


Figure 2: ω_i and Ω . The total mass of workers in both firms is $1 + 1 = 2$. In firm 1 and 2 masses of 0.8 and 0.2 have demanded wages above Ω

Note that our definition of ω implies that *all* workers in the catchment area of a firm, employed *and* unemployed union members vote for the wage. Though this assumption may be not realistic in some cases, it can be shown, that it does not lead to qualitative changes of the main results.¹¹ The meaning of $s_2 = (q_1 + q_2)/2 - s_1$ becomes clear if we write it in the form $s_1 + s_2 = (q_1 + q_2)/2$ which is simply the generalization of $s_1 + s_2 = 1/2$ for $q_1, q_2 \neq 0$.

The common wage, call it Ω depends (through \tilde{s}) on all ξ_i and q_i . We write down the definition here, since it will play a central role in the following

¹¹Lindblom (1949) initiated the so-called ‘Ceshire Cat’ discussion with the hypothesis, that unions have a natural tendency to shrink if unemployed workers leave the union or have no voting rights. This occurs since the least senior workers with preferences for lower wages become unemployed first and the remaining ones will generate additional unemployment in the next bargain by raising wages. This process continues until the union loses bargaining power because of small membership. Blair and Crawford (1984) (c.f. also Farber, 1986) clear this point by arguing that this problem vanishes if union members account for it in an intertemporal utility maximization procedure. Burda (1990) shows the validity of the argument (at least in many realistic situations) in an intertemporal formal model.

sections.

$$\Omega(\xi_1, \xi_2, q_1, q_2) := \omega_1(\xi_1, \tilde{s}(\xi_1, \xi_2, q_1, q_2)).$$

As will be explained below, a general analysis of employment effects of centralization (going without restrictions on the functions ω_i and the q_i) is quite involved. But we obtain intuitive first results already from a special case where the ξ_i are the only source of heterogeneity, i.e. production function parameters are equal and employment pools have equal size ($q_1 := q_2 := 1$ ¹²). Our strategy is simple: We start from a situation where outcomes in central and local bargaining are identical in our setting and generate a ‘perturbation’ by changing one parameter (here: the stochastic shock ξ_1). Then we can use calculus to analyze differences between employment in local and central wage setting. Let

$$\eta_l = \phi(\omega(\xi_1, 1/2)) + \xi_1 + \theta_1 + \phi(\omega(\xi_2, 1/2)) + \xi_2 + \theta_2$$

denote gross employment in a local and

$$\eta_c = \phi(\Omega(\xi_1, \xi_2)) + \xi_1 + \theta_1 + \phi(\Omega(\xi_1, \xi_2)) + \xi_2 + \theta_2$$

in a central wage setting environment. Note that ϕ and ω are not indexed any more, and that we have set $q_1 := q_2 := 1$ implying that the ξ_i are the only remaining source of heterogeneity.

To compute the expected employment levels observe that (in general)

$$\begin{aligned} E[n|\xi] &= \int_{-\infty}^{\infty} \max\{0, z(\xi) + \theta\} dF(\theta) = \int_{-z(\xi)}^{\infty} (z(\xi) + \theta) dF(\theta) \\ &= z(\xi)\{1 - F(-z(\xi))\} + \int_{-z(\xi)}^{\infty} \theta dF(\theta) \\ &= \{1 - F(-z(\xi))\} \{z(\xi) + E[\theta|\theta > -z(\xi)]\} \end{aligned}$$

with shorthand $z(\xi) := \phi(\omega(\xi)) + \xi$. The expression in the last line has an obvious interpretation. The first term in curly braces represents simply the probability that employment is positive and the second one is the expected employment, given employment is positive.

¹²Since s is a (continuous) index, we can set the q_i to unity without loss of generality

The corresponding expected employment levels for local and central employment are then

$$\begin{aligned}
E[\eta_l|\xi_1, \xi_2] &= \sum_{i \in \{1,2\}} \left\{ z(\xi_i) \{1 - F(-z(\xi_i))\} + \int_{-z(\xi_i)}^{\infty} \theta dF(\theta) \right\} \\
E[\eta_c|\xi_1, \xi_2] &= \sum_{i \in \{1,2\}} \left\{ z_i(\xi_1, \xi_2) \{1 - F(-z_i(\xi_1, \xi_2))\} + \int_{-z_i(\xi_1, \xi_2)}^{\infty} \theta dF(\theta) \right\}
\end{aligned}$$

where $z_i(\xi_1, \xi_2) := \phi(\Omega(\xi_1, \xi_2)) + \xi_i$. We evaluate the expected employment difference $E[\eta_c|\xi_1, \xi_2] - E[\eta_l|\xi_1, \xi_2]$ by means of a Taylor series approximation starting from a situation where $\xi_1^0 = \xi_2^0 =: \xi_0$. If (as assumed here) the ξ_i are the only source of heterogeneity, $E[\eta_c] = E[\eta_l]$ in this situation. Since the situation is symmetric ($q_1 = q_2$), it suffices to consider an increase in ξ_1 when holding ξ_2 constant. With local wage setting, only expected employment in firm 1 changes. Then

$$\left. \frac{\partial E[\eta_l]}{\partial \xi_1} \right|_{\xi_i = \xi_0} = \left\{ 1 + \phi'(w_0) \left. \frac{\partial \omega(\xi)}{\partial \xi} \right|_{\xi_i = \xi_0} \right\} \{1 - F(-\phi(w_0) - \xi_0)\}$$

where $w_0 = \omega(\xi_0, 1/2)$. Henceforth we will drop the second argument of $\omega(\cdot)$ for notational convenience if this does not lead to confusion. For central wage setting, wages in both firms increase, but the increase is smaller. We obtain

$$\left. \frac{\partial E[\eta_c]}{\partial \xi_1} \right|_{\xi_i = \xi_0} = \left\{ 1 + 2\phi'(w_0) \left. \frac{d\Omega(\xi_1, \xi_2)}{d\xi_1} \right|_{\xi_i = \xi_0} \right\} \{1 - F(-\phi(w_0) - \xi_0)\}$$

The expressions in large curly braces give the increase in employment, given employment is positive. They split into the direct effect 1 and the indirect effects $\phi' d\omega/d\xi_1$. The factor $1 - F(\cdot)$ accounts for the fact that a marginal increase of ξ_1 has effects only if employment is positive, i.e. if $\theta > -\phi(w_0) - \xi_0$.

Comparison of the both expressions reveals that central wages generate higher (equal/lower) employment than local ones if

$$\left. \frac{\partial \omega(\xi)}{\partial \xi} \right|_{\xi = \xi_0} \begin{matrix} \leq \\ \geq \end{matrix} 2 \left. \frac{d\Omega(\xi_1, \xi_2)}{d\xi_1} \right|_{\xi_i = \xi_0} \quad (5)$$

The interpretation of this condition is straightforward. With local wage setting, only the wage of firm 1 is affected by rise of ξ_1 , whereas with central

wage setting, both firms face the same (but lower) wage increase. We will show below that (5) is met with equality. At a glance one would conclude from this that no centralization effects exist. A closer look, however, reveals that this result were valid only if $\phi(\cdot)$ and $\omega(\cdot)$ were linear functions (since we applied a first order Taylor series expansion until now) and marginal changes only. Thus the effects must be of second order. In reality, the difference $\xi_1 - \xi_2$ may be large, destroying the validity of first order approximations.¹³

Before we proceed with the straightforward but tedious computations, let us pause for a moment to get some intuition for the issues involved. First consider the median wage. If ω is linear in s , the definition of the median wage (4) tells us that (after an increase of ξ_1) the change of $\omega(\xi_1, \xi_2)$ is exactly one half of the change in $\omega(\xi_1)$. If ϕ is linear too, it is clear that centralization has no employment effects. However, if ϕ is convex, centralization must have negative employment effects, since then a wage increase of dw in one firm leads to a smaller employment reduction than the sum of employment reductions from wage increases $dw/2$ in two firms. However, since $\omega(\cdot)$ and $\tilde{s}(\cdot)$ are nonlinear functions too, the median wage increase possibly is smaller than $1/2$ of the local wage increase in firm 1. This may overcompensate the labour demand function effect. However, things are a little bit more complicated, since changes of ξ_1 affects also the truncation (represented by the factor $1 - F(\cdot)$).

To show formally that first order effects vanish, we compute $d\Omega(\xi_1, \xi_2)/d\xi_1$. After substitution of $\Omega(\xi_1, \xi_2) \equiv \omega(\xi_1, \tilde{s}(\xi_1, \xi_2))$ we have

$$\frac{d\Omega(\xi_1, \xi_2)}{d\xi_1} = \frac{\partial\omega}{\partial\xi} + \frac{\partial\omega}{\partial\tilde{s}} \frac{d\tilde{s}}{d\xi_1} \quad (6)$$

$d\tilde{s}/d\xi_1$ is obtained by implicit differentiation of (4)

$$\frac{d\tilde{s}}{d\xi_1} = - \frac{\frac{\partial\omega(\xi_1, s)}{\partial\xi_1}}{\frac{\partial\omega(\xi_1, s)}{\partial s} + \frac{\partial\omega(\xi_2, 1-s)}{\partial s}} > 0 \quad (7)$$

For $s = 1 - s$ and $\xi_1 = \xi_2 =: \xi^0$ this simplifies to

$$\frac{d\tilde{s}}{d\xi_1} = - \frac{\frac{\partial\omega(\xi, s)}{\partial\xi}}{2 \frac{\partial\omega(\xi, s)}{\partial s}} \quad (8)$$

¹³The Existence of second order effects only does not mean that they must be small or ignorable. They are small only at the margin and grow with order proportional to $(\xi_1 - \xi_2)^2$.

After substitution of (6) and (8) into (5) we see that it is an equality.

Now let us investigate the second order derivatives. We have

$$\left. \frac{\partial^2 E[\eta_l]}{\partial \xi_1^2} \right|_{\xi_i = \xi_0} = (1 + \omega_\xi \phi')^2 f + (1 - F) \{ \phi' \omega_{\xi\xi} + \omega_{\xi_1}^2 \phi'' \} \quad (9)$$

$$\begin{aligned} \left. \frac{\partial^2 E[\eta_c]}{\partial \xi^2} \right|_{\xi_i = \xi_0} &= \{ 1 + 2 \phi' \Omega_{(\xi_1)} (1 + \phi' \Omega_{(\xi_1)}) \} f \\ &+ 2 (1 - F) \{ \phi'' \Omega_{(\xi_1)}^2 + \phi' \Omega_{(\xi_1 \xi_1)} \} \end{aligned} \quad (10)$$

where we have dropped all arguments of the functions for notational convenience. Note that whereas ω_h, ω_{hh} denote the first and second *partial* derivatives of ω with respect to h , $\Omega_{(h)}, \Omega_{(h,h)}$ denote the first and second *total* derivatives of Ω with respect to h .

Before we proceed with our investigations let us again pause a moment to interpret the second order derivatives. The terms multiplied by $(1 - F)$ represent the change of the derivative of employment, given positive employment (in the sense that $\phi(w) + \xi + \theta > 0$). The other terms account for the change of the truncation limit $P(\text{employment} > 0)$ due to a shift of ξ_1 .¹⁴

After substitution of $\Omega_{(\xi_1)} = \omega_\xi/2$ the difference of the second derivatives has the form

$$\begin{aligned} \frac{\partial^2 E[\eta_c]}{\partial \xi_1^2} - \frac{\partial^2 E[\eta_l]}{\partial \xi_1^2} &= (1 - F) \phi' \{ 2 \Omega_{(\xi_1 \xi_1)} - \omega_{\xi\xi} - \frac{\omega_\xi^2 \phi''}{2 \phi'} \} \\ &- f \omega_\xi \phi' (1 + \omega_\xi \phi'/2) \end{aligned} \quad (11)$$

Implying that centralization leads to higher employment if

$$(1 - f) \{ 2 \Omega_{(\xi_1 \xi_1)} - \omega_{\xi\xi} - \frac{\omega_\xi^2 \phi''}{2 \phi'} \} < f \omega_\xi (1 + \omega_\xi \phi'/2) \quad (12)$$

This expression shows that the effects of ξ on ‘uncensored’ employment possibly are contrary to the effects on truncation (represented by the right hand side term). Still is not very handy. It will turn out that no clear and unambiguous result can be derived. But we can exploit the formulas, a) to learn

¹⁴A change of ξ causes $P(n_1 > 0)$ to increase by $(1 + \omega_\xi \phi') f$ for local wage setting. Consequently the term $(1 + \omega_\xi \phi')^2 f$ represents the expected change of the derivative due to the effect on truncation. With central wages, different reactions of employment in firm 1 and 2 complicate the situation a little bit. The change in $P(n > 0)$ is $f(1 + \phi' \Omega_{(\xi_1)})$ for firm 1 and $f \phi' \Omega_{(\xi_1)}$ for firm 2.

something about the model properties favouring local or central wages and b) to provide more accessible results for special cases.

As a first special case suppose that truncation (of the distribution of employment) does not occur in the model¹⁵. Then the condition above reduces to $2\Omega_{(\xi_1\xi_1)} - \omega_{\xi\xi} - \omega_{\xi}^2\phi''/(2\phi')$. If we insert

$$\Omega_{(\xi_1\xi_1)} \equiv \frac{d^2\Omega(\xi_1, \xi_2)}{d\xi_1^2} \equiv \frac{\partial^2\omega}{\partial\xi_1^2} + \left\{ 2\frac{\partial^2\omega}{\partial\xi\partial s} + \frac{\partial^2\omega}{\partial s^2} \right\} \frac{\partial\tilde{s}}{\partial\xi} + \frac{\partial\omega}{\partial s} \frac{\partial^2\tilde{s}}{\partial\xi_1^2} \quad (13)$$

and $\omega_s = -\omega_x$ (from equation 18) and assume labour demand to be linear ($\phi'' \equiv 0$), then this expression reduces to

$$\omega_s(\omega_{ss} - 3\omega_{\xi s}) < 0.$$

Substitution from (18) shows that this is met for convex wage setting functions, i.e. $\omega_{ss} > 0$. However, the calculations above have shown that ω_{ss} is a rather complex expression and that a simple relation between ω and the properties of (u, ϕ, F) does not exist. The expression in curly braces in (12) also shows that convexity of the labour demand function is unambiguously in favour of local wages.

Now let us inspect the more general case where truncation occurs. Then the truncation term works in favour of central wage setting if

$$f\omega_{\xi}\phi'(1 + \omega_{\xi}\phi'/2) < 0$$

or $\omega_{\xi}\phi' < -2$. In the case of linear labour demand it is met if

$$\epsilon_1 > \frac{2(gu'' - 2u'\phi'\epsilon_2)(gu'' - u'\phi'\epsilon_2)}{g(u'\phi')^2}$$

where $g := 1 - F$ and $\epsilon_1 > 0$ and $\epsilon_2 > 0$ are shorthands for the expression (3) and the negative of expression (4) respectively. This reduces to $\epsilon_1 > 4\epsilon_2^2/g$ if workers are expected earnings maximizers (i.e. $u' = 1$ and $u'' = 0$). After backsubstitution from (16) this reads

$$f' < -3f^2/g$$

meaning that $\bar{\theta} = s - \phi - \xi$ must be located where f is decreasing.

¹⁵Of course, this implies restrictions on F , u and ϕ . We do not try to state them formally.

Probably it would be possible to squeeze some further results out of the model. But we have good reasons to doubt that they would be worth the effort, since we deal with the simplest version of the model and even here the dependence of employment effects on properties of u , ϕ , and F is intransparent. Furthermore our results are valid only at the margin, i.e. for small differences $\xi_1 - \xi_2$. Therefore we summarize the most important results here and proceed by illustrating the effects in two parametric numerical examples.

As we noted already, if the wage claim function ω is convex in s , $2\Omega_{(\xi_1, \xi_1)} < w_{\xi\xi}$. This makes the left hand side of (12) small and is consequently in favour of central wage setting. The direct consequence of convexity of ω is that the reaction of the central wage to a demand shock ξ in one firm is *less* than half of the reaction of a local wage in one firm (hit by the shock).

The more moderate increase of central wages must overcompensate *at least* the adverse effect (represented by $\omega_\xi^2 \phi'' / (2\phi')$) stemming from convexity of labour demand function. Convexity simply means that (other things equal) a wage increase of two units in one firm reduces employment less than one-unit increases in two (identical) firms.

Finally truncation of the distribution of labour demand (represented by the right hand side of (12) generates a third effect on relative employment. It's sign is ambiguous, however and we see – again from inspection of (12) – that it is negligible if unemployment risks of the median worker are low, since then employment probability g is high and f small.

This seems to be all we can say about the model at a general level. Since it seems hardly possible to consider less restrictive scenarios (e.g. heterogeneity with respect to q or production function parameters) and the analytical results tell us less about the magnitude or relative importance of the effects, we amend our small investigation by a short numerical illustration.

5 Some Numerical Illustrations

Here we evaluate centralization effects using a parameterized model. This allows us to relax some of the restrictions applied above. We confine our analysis to the special case of two firms but relax the assumption $q_1 = q_2$ and consider more general form of heterogeneity of labour demand.

We use the constant relative risk aversion utility function

$$u(w) = w^\beta$$

and a CES production function with fixed capital stock (normalized to unity)

$$h(n) = \lambda \{(\alpha n)^\rho + 1\}^{1/\rho}$$

and labour demand function

$$\phi(w) = \frac{1}{\alpha \left\{ \left(\frac{w}{\alpha \lambda} \right)^{\rho/(1-\rho)} - 1 \right\}^{1/\rho}}$$

λ can be interpreted as total factor productivity or demand shift indicator. θ is assumed to be distributed according to a Weibull distribution with CDF $F(\theta; a, b) = 1 - \exp(-(\theta/b)^a)$. The Weibull distribution is used because of its flexible functional form (with two parameters only) and its simple and closed form CDF representation. Furthermore the Mill's ratio has the simple form $x^{a-1}a/b^a$ and is increasing (as required for single-peakedness) for $a > 1$. Its support is $[0, \infty)$, but we can produce negative shocks to the labour demand by rescaling ξ . The model is solved by numerical optimization and root search procedures.¹⁶ The following figures compile results of four simulations.

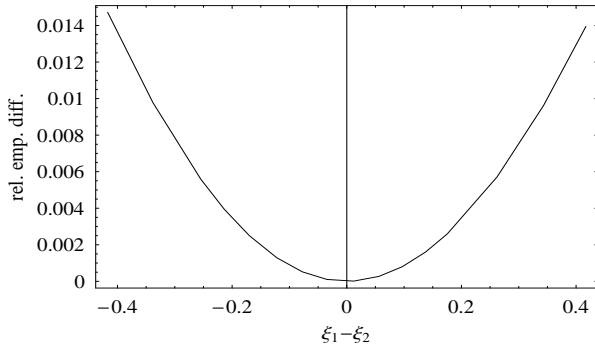


Figure 3: relative employment effect of a change in ξ

All graphs are obtained by varying one parameter, while holding all other constant. The standard parameter values are

| | | | | |
|--------------------------|---------------|----------------|---------------|--|
| probability distribution | $a = 1.5$ | $b = 1.0$ | | |
| utility function | $\beta = 0.3$ | $w_0 = 1$ | | |
| production function | $\alpha = 1$ | $\rho = -2$ | $\lambda = 3$ | |
| other | $q = 1$ | $\xi_0 = -1.5$ | | |

¹⁶In some cases the Newton root search algorithm failed to find the correct solution to (4). Therefore we reformulated the root search problem as a (degenerate) minimization problem and used a robust global minimization algorithm to solve it. We followed this strategy mainly for convenience reasons, since our symbolic mathematics Package (Mathematica) provides convenient and robust global minimization routines.

and relative employment difference $((E[n_c] - E[n_l]) / E[\eta_l])$ is defined such that a positive value implies higher employment with central wage setting. In the first simulation we vary ξ holding all other parameters constant. To interpret the magnitude of the effect correctly, note that the maximum difference of ξ is equal to central employment for $\xi_1 - \xi_2 = 0$. Since this is a rather large difference, the graph shows that significant employment effects (maximum is about 1.5 percent here) occur only if firm size differences are large. However, the scenario appears not unrealistic, since the relative difference of wages at the extreme points is about 6.5 percent. This is rather moderate compared to maximum firm size wage effects of more than 20 percent reported in the empirical literature (c.f. Oi & Idson, 1986 for a survey or Wagner, 1991 for Germany).

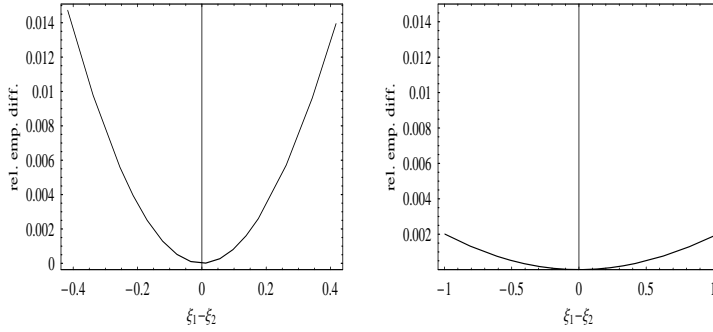


Figure 4: Comparison of nonlinear and (almost) linear case (obtained by setting $b := 3$)

Figure 4 shows that centralization effects vanish if ω approaches a linear function. The right hand side graph was obtained by setting the distribution function parameter $b := 3$ such that $\omega(\cdot)$ becomes (almost) linear.

Figure 5 relates to the case of risk neutral workers. It shows that the centralization effect diminishes, but a maximum effect of about 1 percent remains.

Figure 6 shows that the effects increase *ceteris paribus* to about 2.5 percent if the catchment areas (and thus unemployment rates in the areas) of firms differ. In the simulation q_2 was set to 1.5, i.e. catchment area is 50 percent larger in firm 1. The effects are not symmetric with respect to the difference $\xi_1 - \xi_2$, and it matters whether firm 1 or firm 2 is affected by the change in ξ . As can be seen in the figure, the effect becomes even negative for certain values of ξ but the magnitude is small. Figure 7 contains an example for negative centralization effects. They occur if the labour productivity

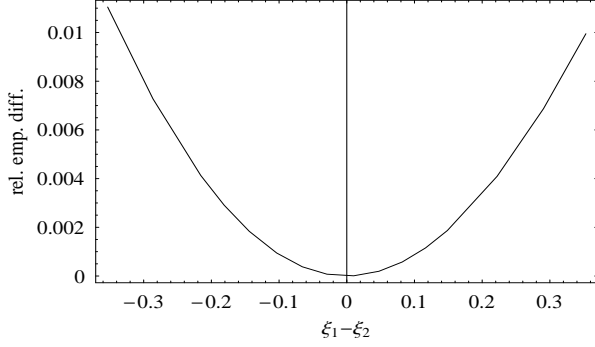


Figure 5: Relative employment effects in the case of risk neutral workers ($\beta = 1$)

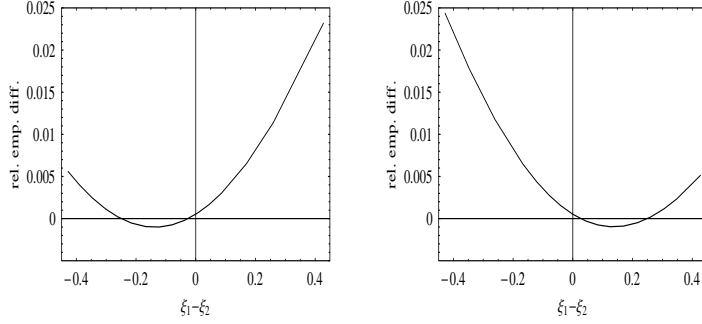


Figure 6: Differences in catchment area sizes ($q_2 = 1.5$). ξ_1 varies in the left hand side and ξ_2 varies in the right hand side.

parameter α varies between firms. These effects are of considerable size for a negative difference $\alpha_1 - \alpha_2 = -0.15$.¹⁷

The last figure illustrates the positive but small effects of a change in the total factor productivity parameter λ .

To sum up, we find positive as well as negative employment effects. However, considering the simple structure of the model, it appears less promising to ‘estimate’ the model empirically.

¹⁷We have to note that the computations become unstable for larger differences of α .

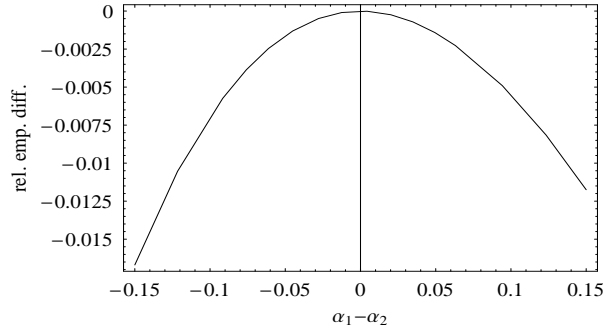


Figure 7: Employment effects associated with a change of α .

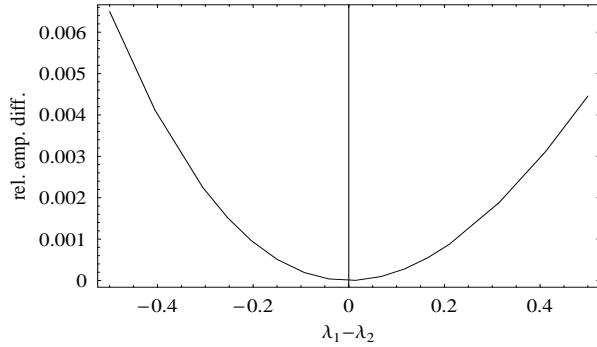


Figure 8: Employment effects associated with a change in λ .

A Qualification

The probably most severe shortcoming of our model is the restriction to the case of two firms only. Since the generalization of the model with respect to the number of firms promises to be complicated and tedious, we are content with some speculations here. If a large number of firms in our economy is homogenous (with respect to labour demand characteristics) and only a few firms deviate, the relative importance of the deviating firms in wage setting and employment becomes small and we expect that centralization effects are negligible. Even in an economy with heterogenous firms, the effects shrink if we have a continuum of firm sizes. Consequently our results tend to overestimate the real effects.

6 An even Simpler Version of the Model with a Closed Form Solution

By putting additional constraints on the production function we can construct a version of the model with a closed form solution. Though is is even less realistic than the model above, its results can be verified directly (i.e. without use of an intransparent numerical simulations). Again we employ a constant relative risk aversion (CRRA) utility function

$$u(W) = W^\beta$$

and a Cobb-Douglas type production function $g(N) = \gamma N^\alpha$ with labour demand function

$$\phi(W) = \left(\frac{W}{\alpha\gamma}\right)^{1/(\alpha-1)} \quad \text{or} \quad \ln[\phi(w)] = \psi - \eta w$$

where $\psi = \ln(\alpha\gamma)/(\alpha - 1)$ and $\eta = 1/(1 - \alpha)$. In order to obtain model with a closed form solution we have to make some further restrictions and normalizations. We set $b := 0$ (to eliminate the alternative income term) and choose a multiplicative exponential specification for the terms ξ and θ . Then $E[U]$ has the form

$$\begin{aligned} E[U(W|S)] &= W^\beta P[S \leq \phi(w)e^\xi e^\theta] \\ &= W^\beta \{1 - P[s > \ln(\phi(W)) + \xi + \theta]\} \end{aligned} \tag{14}$$

where lower case letters denote logs of the corresponding (latin) capital letters. After further manipulation we arrive at

$$\ln E[U(w|s)] = \beta w + \ln [1 - F(s + \eta w - \psi - \xi)] \tag{15}$$

where $F(\cdot)$ is the CDF of θ . If θ has a uniform distribution in the range $[0; 1]$, $f(\theta) = 1$ and $F(\theta) = \theta$ and the (log) worker utility function has the simple form

$$\ln(E[U|s, w, \xi]) = \beta w + \ln \left[s - \xi - \frac{w}{1 - \alpha} + \frac{\ln(\alpha\gamma)}{1 - \alpha} \right]$$

Solving the worker's utility maximization problem gives

$$\omega(s, \xi) = (s - \xi)(1 - \alpha) - 1/\beta + \ln(\alpha\gamma)$$

$\Omega(\xi_1, \xi_2)$ now has a closed form representation.

6.1 Effects of Changes in ξ

If firms and workers are identical with respect to all parameter values except ξ , \tilde{s} has the form

$$\tilde{s} = (q_1 + q_2)/4 + (\xi_1 - \xi_2)/2$$

and the median wage is

$$\Omega = \ln(\alpha\gamma) - 1/\beta + (1 - \alpha)\{(q_1 + q_2)/2 - (\xi_1 + \xi_2)\}/2$$

After setting $q_1 = q_2$ and some further straightforward manipulations we can write the expected employment difference between central and local wage setting as

$$E[\eta_c] - E[\eta_l] = (e - 1) e^{\frac{2-\beta(\alpha-1)}{2\beta(\alpha-1)}} \left\{ e^{\frac{-\xi_1}{2} - \frac{\xi_2}{2}} (e^{\xi_1} + e^{\xi_2}) - 2 \right\}$$

This expression is positive if the expression in curly braces is. It is easy to show that it is zero iff $\xi_1 = \xi_2$ and strictly positive otherwise. Note that only the magnitude but not the sign does depend on α and β . α and β have *cet. par.* a negative and positive effect on its magnitude¹⁸ However, figure 9 shows, that the effects are ignorably small. The figure is obtained by holding $\xi_2 = 0.5$ constant and varying ξ_1 in the interval $[0.4; 0.6]$ (note that the relative change of ξ_1 is large, since the central employment level is about 0.1 in our example). In the general case with differing worker pool sizes ($q_1 \neq q_2$), it can be shown that the effect of centralization on employment is ambiguous.

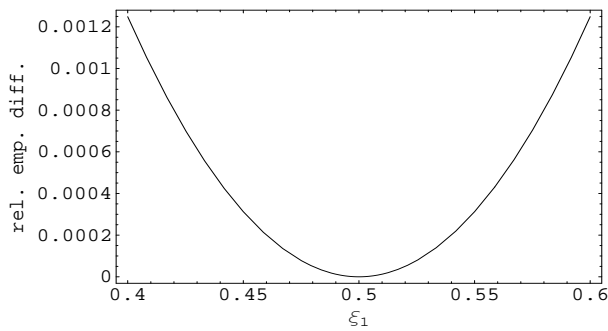


Figure 9: Relative employment effects of a change in ξ

¹⁸The derivatives of the exponent with respect to α and β are $\frac{-1}{(a-1)^2 b}$ and $\frac{1}{b^2 (1-a)}$.

6.2 Effects of Changes in γ

Now consider the impact of variability in γ on employment. With $\xi_i := 0$ the general solution is

$$\tilde{s} = -\frac{-\ln(\alpha_1 \gamma_1) + \ln(\alpha_2 \gamma_2) + \frac{1}{2}(q_1 + q_2) - \frac{1}{2}(q_1 + q_2)\alpha_2 + \frac{1}{\beta_1} - \frac{1}{\beta_2}}{\alpha_1 + \alpha_2 - 2}$$

with further parameter equality restrictions (including $q_1 = q_2 = 1$) we obtain

$$\tilde{s} = \frac{\alpha - 1 + \ln(\gamma_1) - \ln(\gamma_2)}{2(\alpha - 1)}$$

After further manipulations we obtain an algebraic expression for the employment difference

$$\sqrt{e}(e-1)e^{\frac{1}{\beta(\alpha-1)}}(\gamma_1\gamma_2)^{\frac{1}{2(1-\alpha)}}\left\{\gamma_1^{\frac{1}{2(\alpha-1)}} - \gamma_2^{\frac{1}{2(\alpha-1)}}\right\}^2.$$

Again it is immediately clear that employment differences are positive for all $\gamma_2 \neq \gamma_1$ and zero otherwise. However, the implied effects are considerably large as figure 10 shows. It is generated by varying γ_2 in the interval $[0.7, 1.0]$ holding $\gamma_1 = 1.0$ constant.

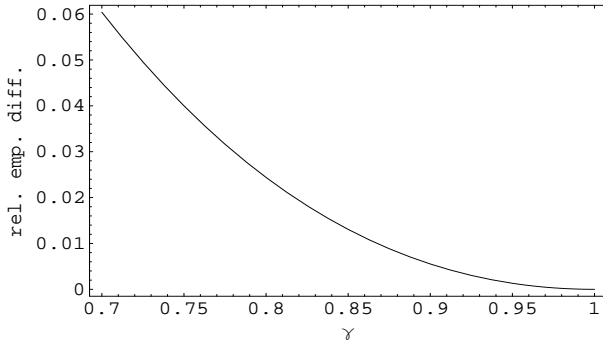


Figure 10: Relative employment effects of a change in γ

7 Conclusion

In our stylized median voter model we found ambiguous employment effects of centralization in wage setting. Considering the information requirements

of the model it seems to be at least difficult (and probably impossible) to predict the effects on the base of available empirical data and parameters. However, simple reasoning suggests that the effects found here shrink and may become negligible if the number of firm increases.

The most legitimate criticism of our model is its shortcoming of realism. Therefore we should conclude with a thorough discussion of its central assumptions. Before we do this, we once more note that the the aim of the model is not to derive guidelines for politicians, unions and firms, but to show that employment effects of institutional changes in wage setting may be driven by combinations of parameters making it extremely difficult to predict the direction of changes.

First, our model shares a crucial shortcoming with most other investigations of centralization. It takes the degree of centralization as given and does not explain why unions (and employers) choose the one or the other bargaining regime. Many papers find that centralized wage setting internalizes externalities and therefore should be efficient (at least if the considered externalities were the only issue in bargaining), but they don't explain why wage setting remains local in many countries even if unions and employers were free to centralize. Most authors will respond to this critique with the custom argument that economic models isolate some certain aspects of reality and ignore all other in order to simplify things and that some of the ignored aspects (for example firm heterogeneity or institutional costs) may be responsible for the observed stability. This view assumes tacitly that the these aspects are neutral with respect to employment – a rather heroic assumption.

We note that the degree of centralization could in principle be endogenized in our framework, with a clear and simple answer: In the special case of two firms it is clear that wages would be set locally if θ were the only stochastic parameter, since the work force of a firm will allways prefer its own local median to an aggregated one (by definition). If other parameters are stochastic, at least in the medium and long run (for example ξ labour demand parameters), then central wages are possible with risk averse workers, since the central wage is less volatile than local ones. However, this result is not likely, since firm heterogeneity appears to be a phenomenon with high persistence.

Thus uncertainty alone cannot explain the existence and stability of central wage setting regimes and we have to look for other arguments. The best candidates to fill this gap are bargaining costs, institutional barriers or deviations from the standard utility independence assumptions. Fixed bargaining costs have the 'advantage' that they can be introduced into the model with-

out any other adaptations. However, we fear that other institutional issues and deviations from standard utility theory are more important. But it is more difficult to tackle them in a formal model. Furthermore one then has to accept that the results may be driven by arbitrary assumptions since it is hard to find clear evidence on the importance of such factors (e.g. framing, fairness and adherence to norms, cf. Fehr and Schmidt (2000) for a survey).

Finally, the median voter mechanism relies on single-peakedness of utility functions. As is now well known from the literature on social welfare and voting equilibria, a unique voting equilibrium may not exist at all if workers vote on different variables (e.g. wages and working time) simultaneously. Even if this ‘curse of dimensionality problem’ were neglectible, we had to exclude any other institutional imperfections (e.g. manipulation of workers by union leaders) in order to put the median voter results on save grounds.

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Signs of the Second Order Derivatives of ω

The following derivation shows that signs of the second order derivatives of ω are undetermined without further restrictions on f , ϕ and u . We substitute the following restrictions from the first order condition (1) and the single-peakedness conditions (3) and (4) into $d^2\omega/ds^2$

$$\begin{aligned} u(w) - u(b) &= -\frac{g}{f} \frac{u'}{\phi'} \\ f' &= f^2/g - \epsilon_1 \\ f &= \frac{g}{\phi'} \underbrace{\left\{ \frac{\phi''}{\phi'} - \frac{u''}{u'} \right\}}_{=:k} + \epsilon_2 \end{aligned} \tag{16}$$

with shorthand $g := 1 - F$ and $\epsilon_1 > 0$ and $\epsilon_2 > 0$ are shorthands for the expression (3) and the negative of expression (4) respectively. After tedious manipulations we end up with the lengthy expression

$$\begin{aligned} \frac{\partial^2 \omega}{\partial s^2} &= \frac{u' \phi'^3}{D} \left\{ (\epsilon_2 + g k / \phi')^4 \phi' u'' \right. \\ &\quad + g (\epsilon_2 + g k / \phi')^2 \phi' \left\{ u' \phi' f'' + 2 \left((\epsilon_2 + g k / \phi')^2 / g - \epsilon_1 \right) u'' \right\} \\ &\quad + \frac{u' ((g k + \epsilon_2 \phi')^2 - g \epsilon_1 \phi'^2)^2 \phi''}{\phi'^4} - g (\epsilon_2 + g k / \phi') \times \\ &\quad \left. \left\{ u' \left(\epsilon_1 - (\epsilon_2 + g k / \phi')^2 / g \right)^2 \phi'^2 - g \phi' f'' u'' + g u' f'' \phi'' \right\} \right\} \end{aligned} \tag{17}$$

where

$$D = \left\{ g \epsilon_1 u' \phi'^3 - (g k + \epsilon_2 \phi') \left\{ 3 \epsilon_2 u' \phi'^2 + g (3 k u' \phi' + \phi' u'' - u' \phi'') \right\} \right\}^2$$

whose sign is not determined without further restrictions on f , ϕ and u . Consequently the sign of $\partial^2 \omega / \partial s^2$ (and thus $\partial^2 \omega / \partial \xi^2$) is not determined. It is easy to see this if we consider a special case of the model with risk neutral workers and linear labour demand function. In this case $\partial^2 \omega / \partial s^2$ simplifies to

$$\frac{\epsilon_2 \{g(g(f'' \epsilon_2 - \epsilon_1^2) + 2 \epsilon_1 \epsilon_2^2) - \epsilon_2^4\}}{g \phi' (g \epsilon_1 - 3 \epsilon_2^2)^2}$$

Note however, that we know that

$$\frac{\partial^2 \omega}{\partial s^2} = -\frac{\partial^2 \omega}{\partial \xi \partial s} \quad \text{and} \quad \frac{\partial^2 \omega}{\partial s^2} = \frac{\partial^2 \omega}{\partial \xi^2} \quad (18)$$

because ξ and s enter all subexpressions of $E[U]$ with opposite sign.

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